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TECHNICAL NOTE

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Effective Modulus for Crack Size Measurement With SE(T) Specimens using Unloading Compliance

Reference

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ABSTRACT

Low-constraint toughness tests have been increasingly used in pipeline flaw assessment. A number of protocols have been published using a test employing a clamped single-edge tension (SE(T)) specimen with the distance between the clamps H equal to ten times the specimen width W, and specimen width W equal to thickness B. For this geometry, the constraint over the bulk of the specimen varies between plane stress and plane strain. This affects the elastic compliance, commonly used to estimate crack size. It was the intent of the present work to assess the effects of constraint on the compliance and to identify the best combination of compliance equation and modulus to estimate crack size from CMOD compliance where CMOD is the crack mouth opening displacement. To do this, values of compliance from finite element analyses were provided by the authors of this paper from three separate laboratories. 2D plane strain, 3D plane-sided, and 3D side-grooved specimens were analyzed. The data were used to assess available compliance equations and to propose a new one for the specific geometry of interest in this work.

Keywords

low-constraint toughness testing, single-edge tension SE(T), single-edge-notch tension (SENT), crack size measurement, CMOD unloading compliance

Nomenclature

a = crack size
B = specimen thickness
B_{eff} = effective thickness, $B_{eff} = B-(B-B_N)^2/B$
B_N = net thickness, $B_N = B$-(total side groove depth)
C = CMOD compliance, $\Delta CMOD/\Delta P$
CMOD = crack mouth opening displacement
E = Young's modulus of elasticity
ECA = Engineering Critical Assessment
E_{eff} = effective elastic modulus
$E_{pl\varepsilon} = E/(1 - \nu^2)$
$E_{pl\sigma} = E$
FEA = finite element analysis
$f(u)$ = compliance equation giving a/W as a function of u
H = distance between grips ("daylight")
J = J-integral
J-R = J resistance
P = force (load)
SE(B) = single-edge bend
SE(T) = single-edge tension (sometimes denoted SENT for single-edge-notch tension)
$u = 1/(1+\sqrt{(B_{eff}CE_{eff})})$
W = specimen width
α = constraint parameter, $\alpha = (E_{eff} - E_{pl\sigma})/(E_{pl\varepsilon} - E_{pl\sigma})$
ν = Poisson's ratio

Introduction

To perform engineering critical assessment (ECA) of a flaw in a pipeline, it is necessary to measure the toughness. For surface flaws, it is well recognized that traditional techniques to measure the size-independent material toughness are highly conservative in this application and may unnecessarily penalize the material. It is preferable to use test techniques that better reproduce the actual constraint at the tip of the flaw. Consensus has been building on the use of single-edge tension (SE(T) (also referred to as single-edge-notch tension, SENT) specimens for this purpose. The first standard to emerge was DNV-RP-F108 [1], that recommended a geometry with $H/W = 10$, where H is the distance between the grips and W is the specimen width, and $B/W = 2$, where B is the specimen thickness, although other geometries ($1 \le B/W \le 5$) were allowed. The specimens were not side-grooved, and a multi-specimen procedure was used to generate J-R curves. More recently, single-specimen test procedures [2,3] have been developed in which the crack size is monitored by unloading compliance or electric potential drop techniques.

The use of the term "constraint" requires some discussion. It is used in fracture toughness testing to describe the degree of triaxiality at a given location in a specimen, notably in the vicinity of a crack tip. Triaxiality is highest near the crack tip, where the stress gradients are high and Poisson contractions in the direction of the crack front are restricted. However, at the free surfaces of the specimen, the stress normal to the surface is necessarily zero. It follows that triaxiality and constraint is smaller for a thin specimen than for a thick one. Similarly, for a plate-like or rectangular specimen, remote from a crack tip or other stress concentrator, the through-thickness constraint is small, hence approximating a state of plane stress. Consequently, in a typical fracture toughness test specimen, the degree of constraint varies from near-plane-strain to near-plane-stress depending on the location in the specimen. Modelling such a specimen assuming either plane strain or plane stress conditions in the calculations generates only an approximation to the actual stress state, giving either an upper or lower bound to the mechanical variables. The selection of an appropriate bound for calculations to represent the condition of a particular test specimen, i.e., a plane stress or plane strain state, is a central problem in applying FEA in fracture mechanics.

The unloading compliance technique for measurement of crack size incorporates a relationship between crack size and compliance, most conveniently the CMOD compliance where CMOD is the crack mouth opening displacement:

$$a/W = f(u) \quad (1)$$

with

$$u = g(B_{eff}CE_{eff}) \quad (2)$$

where:

C = the CMOD compliance ($C = \Delta CMOD/\Delta P$), and

B_{eff}, E_{eff} = the "effective" thickness and elastic modulus, respectively.

B_{eff} depends on side-groove depth, and E_{eff} depends on specimen constraint. We shall call $f(u)$ the "compliance equation," and BCE the "normalized compliance." Note that it is necessary for accurate estimation of a/W to correct for rotation that occurs during loading as a result of plastic deformation, primarily in the ligament, and causes the load line to move toward the centre of the ligament [4]; however, this will not be discussed in detail here. A number of equations for $f(u)$ have been proposed in the literature, as reviewed recently by Wang and Omiya [5] and Huang and Zhou [6]. After detailed assessment, Wang and Omiya recommended an equation published by Cravero and Ruggieri [7], whereas Huang and Zhou favored an equation from John and Rigling [8]. However, neither Refs. [7] nor [8] contained the data for compliance as a function of a/W upon which the equations were based, thus it is difficult to judge their accuracy. In fact, there is little difference between the results of the published equations provided they are treated as originally intended. This refers in particular to the choice of effective elastic modulus E_{eff}. In some instances the "plane stress

TABLE 1 Results of FEA of 2D plane strain specimens from three independent sources (A), (B), (C); the variable C in $B_{\text{eff}}CE_{pl\varepsilon}$ is the 2D plane strain CMOD compliance and the fifth column reports the average of the values in columns 2 to 4.

2D Plane Strain	$B_{\text{eff}}CE_{pl\varepsilon}$			
	Source			
a/W	(A)	(B)	(C)	Average
0.05			0.295	0.295
0.1	0.609	0.614	0.613	0.612
0.2	1.393	1.402	1.403	1.399
0.3	2.568	2.555	2.560	2.561
0.4	4.315	4.350	4.366	4.344
0.5	7.169	7.233	7.277	7.226
0.6	11.801	11.910	12.018	11.910
0.7	19.269	19.392	19.642	19.434
0.8	30.573	30.768	31.295	30.879
0.9		46.423	47.375	46.899

TABLE 2 Results of FEA for 3D plane-sided specimens from three independent sources (A), (B), (C); the variable C in $B_{\text{eff}}CE_{pl\varepsilon}$ is the 3D CMOD compliance of plane-sided specimens and the fifth column reports the average of the values in columns 2 to 4.

3D Plane-Sided	$B_{\text{eff}}CE_{pl\varepsilon}$			
	Source			
a/W	(A)	(B)	(C)	Average
0.05			0.295	0.295
0.1	0.618	0.624	0.621	0.621
0.2	1.445	1.456	1.453	1.451
0.3	2.665	2.691	2.693	2.683
0.4	4.566	4.613	4.622	4.600
0.5	7.607	7.694	7.723	7.675
0.6	12.547	12.698	12.775	12.674
0.7	20.559	20.761	20.935	20.752
0.8		33.189	33.569	33.379
0.9		50.568	51.181	50.874

modulus," E, is preferred, while in others the "plane strain modulus," $E/(1-\nu^2)$), is used[7]. The modulus is sometimes written as E', with $E'=E$ for plane stress and $E'=E/(1-\nu^2)$ for plane strain. We shall refer to these moduli as $E_{pl\sigma}$ and $E_{pl\varepsilon}$, respectively. Indeed, for the test geometry analyzed in this work – a clamped specimen with square B by B cross section, where B is the specimen thickness – the compliance of a three-dimensional (3D) specimen lies between that of a two-dimensional (2D) specimen in either plane stress or plane strain. This is not a unique situation; for example, in early editions of ASTM E1820-15a [9], the use of $E_{pl\varepsilon}$ was recommended in the compliance equation for single-edge bend (SE(B)) specimens, but this was recently changed to $E_{pl\sigma}$. The actual state of constraint in a 3D SE(B) specimen has been investigated by Wang et al. [10] and by Shen et al. [11], with the latter introducing the idea of using an "effective modulus" to account for the fact that the constraint in 3D specimens lies between plane stress and plane strain.

It is the intent of the present work to assess the effects of constraint in a clamped SE(T) specimen with $H/W=10$ and $W=B$, where H is the distance between the clamps, and W and B are the specimen width and thickness, respectively, which is the most popular geometry currently being studied for standardization. The effect of side grooves on compliance is also assessed, since side grooves are used in practice to maintain crack-front straightness during crack growth. The objective is to suggest the best combination of compliance equation and modulus to estimate crack size from CMOD compliance. To do this, values of compliance using finite element analysis (FEA) have been provided by the authors from three separate laboratories and the data have been used to assess available compliance equations. 2D plane strain, 3D plane-sided, and 3D side-grooved specimens have been analysed[8].

Compliance Data

FEA calculations were performed at the authors' laboratories using their preferred software. A variety of software was used, including Abaqus, ADINA, and WARP3D. Standard procedures were used in elastic FEA. The results are not expected to be sensitive to details of the procedures because elastic FEA calculations are well established. Indeed, mesh refinement would be important if details of the crack-tip stress distribution were to be reported, which is not the case here. Also, no restrictions were placed on side-groove geometries (notch root radius and included angle) apart from depth, in conformity with test practice; standards typically allow a range of side-groove geometries. Indeed, it has been found that the side-groove root radius in particular has a strong influence on the shape of a growing crack. However, it is not expected that these variables would have a significant effect on the CMOD compliance of a straight-fronted crack as assumed in this work, and this is borne out by the fact that FEA results from all laboratories were remarkably consistent.

[7]It is straightforward to demonstrate, for a tensile specimen loaded in the y direction, that the effective modulus $\sigma_{yy}/\varepsilon_{yy}$ is E for an unconstrained (plane stress) specimen and $E/(1-\nu^2)$ for a specimen that is constrained to be in plane strain in the x (or z) direction.

[8]The values assumed for the elastic modulus E differed between the laboratories, ranging from 200 to 210 GPa (bracketing the value of 205 GPa at room temperature recommended by the ASME Boiler and Pressure Vessel Code). This has no effect on the values of Eq 2 because an increase in modulus is compensated by a decrease in compliance and the product remains constant.

TABLE 3 Results of FEA for 3D specimens, plane-sided (3D pl-sided) and side-grooved to a total side-groove depth (% of the width) of 10 % (3D-sg 10 %), 15 % (3D-sg 15 %)and 20 % (3D-sg 20 %) from two independent sources (A), (C); the variable C in $B_{eff}CE_{pl\varepsilon}$ is the 3D CMOD compliance.

3D	$B_{eff}CE_{pl\varepsilon}$				
	Source				
	(A)			(C)	
a/W	3D pl-sided	3D-sg 10 %	3D-sg 20 %	3D pl-sided	3D-sg 15 %
0.05				0.295	0.288
0.1	0.618	0.614	0.603	0.621	0.611
0.2	1.445	1.443	1.437	1.453	1.452
0.3	2.665	2.670	2.676	2.693	2.710
0.4	4.566	4.583	4.606	4.622	4.672
0.5	7.607	7.643	7.687	7.723	7.825
0.6	12.547	12.610	12.668	12.775	12.945
0.7	20.559	20.646	20.661	20.935	21.156
0.8				33.569	33.812
0.9				51.181	51.156

2D PLANE STRAIN AND 3D PLANE-SIDED SPECIMENS

Table 1 shows the results (values of normalized CMOD compliance, $B_{eff}CE_{pl\varepsilon}$, for $0.05 \leq a/W \leq 0.9$) from the three sources ((A),(B), (C)) for the case of 2D plane strain (note that in this case B_{eff} is, of course, B). The agreement is good, with the average absolute error for the cases where all three laboratories provided data ($0.1 \leq a/W \leq 0.8$), equal to 0.50 %.

Table 2 shows results for the 3D plane-sided specimens. Again, agreement is good, with the average absolute error for $0.1 \leq a/W \leq 0.7$ equal to 0.48 %.

The values of the normalized compliance for the 2D plane strain case differ from those of the 3D case, as expected. The difference increases as a/W increases. We shall comment on this later in the context of a transition from primarily plane strain at small a/W to primarily plane stress at large a/W.

EFFECT OF SIDE GROOVES

Table 3 shows results for 3D plane-sided and side-grooved specimens from the two laboratories that provided data for side-grooved specimens. In this case, the thickness has been adjusted using the relation (as found in ASTM E1820-15a [9]):

$$B_{eff} = B - (B - B_N)^2/B \qquad (3)$$

The thickness adjustment is highly effective, the average difference (in absolute values) in $B_{eff}C$ for plane-sided and side-grooved specimens being only 0.70 %. This is consistent with the conclusions of Donato and Moreira [12], who showed that the effective thickness calculated from Eq 3 is very accurate for clamped SE(T) specimens with $H/W = 6$ and $W/B = 2$.

Effective Modulus

As noted above, the effective modulus depends on specimen constraint. It is well known that the ratio between the 2D plane strain and plane stress compliances is simply $E_{pl\sigma}/E_{pl\varepsilon}$, as discussed in detail by Tyson et al. [13]. That is, $C_{pl\sigma}E_{pl\sigma} = C_{pl\varepsilon}E_{pl\varepsilon}$. In other words, at the same value of a/W, the normalized compliance values in the limits of plane strain and plane stress, i.e., $BC_{pl\varepsilon}E_{pl\varepsilon}$ and $BC_{pl\sigma}E_{pl\sigma}$, respectively, are the same. This enables the definition of an “effective modulus” for the 3D case for a given a/W, i.e., $B_{eff}CE_{eff} = B_{eff}C_{pl\varepsilon}E_{pl\varepsilon} = B_{eff}C_{pl\sigma}E_{pl\sigma}$. Any of these three parameters, when used to calculate u from Eq 2 above, should yield the same value of a/W from Eq 1. It has been found convenient, for curve-fitting purposes, to use for the function g of Eq 2 the following equation:

TABLE 4 Values of effective modulus (normalized by E), E_{eff}/E, derived from compliances for 2D plane strain (average values from **Table 1**) and 3D plane-sided specimens (average values from **Table 2**), and constraint parameter α. The fifth column reports the average of the values in columns 2 to 4, and the seventh column reports the average of the values in the sixth column for a/W values between 0.1 and 0.7.

3D Plane-Sided	E_{eff}/E				α	
	Source					
a/W	(A)	(B)	(C)	Average		Average $0.1 \leq a/W \leq 0.7$
0.05			1.097	1.097	0.98	
0.1	1.083	1.081	1.084	1.083	0.84	0.47
0.2	1.059	1.058	1.061	1.059	0.60	
0.3	1.059	1.043	1.044	1.049	0.49	
0.4	1.039	1.036	1.038	1.038	0.38	
0.5	1.036	1.033	1.035	1.035	0.35	
0.6	1.034	1.031	1.034	1.033	0.33	
0.7	1.03	1.026	1.031	1.029	0.29	
0.8		1.019	1.024	1.022	0.22	
0.9		1.009	1.017	1.013	0.13	

TABLE 5 Values of a/W for 2D plane strain specimens calculated from alternative compliance equations, showing the differences from the 2D plane strain (average) results in **Table 1**. The fourth and seventh columns report the averages of the values in the third and sixth columns, respectively, for a/W values between 0.1 and 0.7.

	a/W from Cravero and Ruggieri Eq 6			a/W from John and Rigling Eq 7		
	2D Plane Strain Specimens					
a/W FEA	Using $E_{pl\varepsilon}$	Difference (%)	Average Difference (%) $0.1 \le a/W \le 0.7$	Using $E_{pl\varepsilon}$	Difference (%)	Average Difference (%) $0.1 \le a/W \le 0.7$
0.05	0.03567	−28.67		0.03976	−20.47	
0.1	0.10034	0.34	0.40	0.09945	−0.55	−0.24
0.2	0.20080	0.40		0.20066	0.33	
0.3	0.30197	0.66		0.29952	−0.16	
0.4	0.40162	0.41		0.39915	−0.21	
0.5	0.50169	0.34		0.49951	−0.10	
0.6	0.60202	0.34		0.59714	−0.48	
0.7	0.70244	0.35		0.69659	−0.49	
0.8	0.79914	−0.11		0.80144	0.18	
0.9	0.88686	−1.46		0.89783	−0.24	

$$g(B_{eff}CE_{eff}) = 1/(1 + \sqrt{(B_{eff}CE_{eff})})(= u) \quad (4)$$

It follows from the above that the effective modulus can be derived for the 3D case using $E_{eff}/E_{pl\varepsilon} = C_{pl\varepsilon}/C$ or, equivalently, $E_{eff}/E = (E_{pl\varepsilon}/E)(C_{pl\varepsilon}/C)$. This relation has been used, assuming $\nu = 0.3$, so that $E_{pl\varepsilon} = E/(1-\nu^2) = E/0.91$, to calculate the values of E_{eff}/E shown in **Table 4**. It is convenient to define a parameter α such that $\alpha = 0$ and 1 for conditions of plane stress and plane strain, respectively, i.e.:

$$\alpha = (E_{eff} - E_{pl\sigma})/(E_{pl\varepsilon} - E_{pl\sigma}) \quad (5)$$

Values of α are reported in **Table 4**. Note that the values of α decrease systematically as a/W increases, implying a transition in constraint from primarily plane strain to primarily plane stress. In the region of greatest interest, i.e., $0.1 \le a/W \le 0.7$, the average value of α is slightly less than 0.5, implying that in this range of a/W, the effective modulus is closer to plane stress than to plane strain.

Alternative Compliance Equations

As noted above, there is some disagreement as to whether the compliance equation of Cravero and Ruggieri [7] or that of John and Rigling [8] is more accurate. A comparison of the two equations is made in **Table 5**. The values of a/W have been calculated using the average values of $B_{eff}C_{pl\varepsilon}E_{pl\varepsilon}$ for 2D plane strain specimens (**Table 1**) in the respective compliance equations:

from Cravero and Ruggieri [7]:

$$a/W = 1.6485 - 9.1005u + 33.025u^2 - 78.467u^3 + 97.344u - 47.227u^5 \quad (6)$$

and from John and Rigling[9] [8]:

$$a/W = -0.41881(1/u) + 0.48575(1/u)^2 - 0.16556(1/u)^3 + 0.027639(1/u)^4 - 0.0022859(1/u)^5 + 0.000074897(1/u)^6 \quad (7)$$

If the compliance equations were 100 % accurate, the values of a/W calculated from them would yield the exact values of a/W used in the finite element calculations, i.e., the difference would be zero (as it would also be if the plane stress values of BCE, i.e. $BC_{pl\sigma}E_{pl\sigma}$, were used in comparison with the 2D plane stress data). The differences in the third and sixth columns of **Table 5** reflect the accuracy of the two equations in giving the correct values of a/W using the average values of the normalized compliance $BC_{pl\varepsilon}E_{pl\varepsilon}$ found from the FEA calculations (**Table 1**). Both Eqs 6 and 7 are quite accurate, with the former yielding slight overestimates of a/W and the latter slight underestimates in the range $0.1 \le a/W \le 0.7$.

We have seen in **Table 4** that the effective modulus for 3D plane-sided specimens in the a/W range of most interest is slightly closer to plane stress than to plane strain, implying that the preferred normalized compliance should be $B_{eff}CE_{pl\sigma}$. We turn now to the results of using these calibration equations with $E_{eff} = E_{pl\sigma} = E$ in the normalized compliance $B_{eff}CE_{pl\sigma}$, with C being the 3D (FEA) specimen compliance. The results are

[9]Note that because the individual terms in the right-hand side of this equation are larger than the resultant number in the left-hand side (i.e., a/W is calculated as a small difference between large numbers), it is vital to retain a sufficient number of significant figures in the coefficients on the right-hand side. Unfortunately, in the paper of Huang and Zhou [6] the coefficients in the last two terms were truncated. It is necessary in the John and Rigling equation to retain at least five significant figures in the coefficients as in Eq 6.

TABLE 6 Values of a/W for 3D specimens calculated from alternative compliance equations with effective modulus $E_{eff} = E_{pl\sigma} = E$, showing the differences from the 3D results in **Table 2**. The fourth and seventh columns report the averages of the values in the third and sixth columns, respectively, for a/W values between 0.1 and 0.7.

	a/W from Cravero and Ruggieri Eq 6			a/W from John and Rigling Eq 7		
	3D Plane-Sided Specimens					
a/W FEA	Using $E_{pl\sigma}$	Difference (%)	Average Difference (%) $0.1 \le a/W \le 0.7$	Using $E_{pl\sigma}$	Difference (%)	Average Difference (%) $0.1 \le a/W \le 0.7$
0.05	0.02663	−46.75		0.03383	−32.33	
0.1	0.09291	−7.09	−2.40	0.09173	−8.27	−3.06
0.2	0.19226	−3.87		0.19231	−3.84	
0.3	0.29337	−2.21		0.29104	−2.99	
0.4	0.39446	−1.38		0.39193	−2.02	
0.5	0.49492	−1.02		0.49279	−1.44	
0.6	0.59550	−0.75		0.59085	−1.52	
0.7	0.69649	−0.50		0.69044	−1.37	
0.8	0.79568	−0.54		0.79762	−0.30	
0.9	0.88415	−1.76		0.89455	−0.61	

shown in **Table 6**. Both equations underestimate the crack size, by a few % in the range $0.1 \le a/W \le 0.7$. In this case, the equation of Cravero and Ruggieri is somewhat better than that of John and Rigling. This is consistent with the results of **Table 5**; using the plane stress modulus (E) has made the underestimates of John and Rigling even more marked, but the decrease in a/W resulting from the use of the plane stress modulus has compensated for the overestimated values (when using the plane strain modulus) of Cravero and Ruggieri. The conclusion from **Table 6** must be that the Cravero and Ruggieri equation is to be preferred when the plane stress modulus is used.

However, it is evident from **Table 6** that even using the preferred equation (of Cravero and Ruggieri) there is still an average difference (underestimate) of 2.4 % when using the plane stress modulus to calculate a/W of 3D plain-sided specimens. If the plane strain modulus were used instead, the corresponding average errors would be 2.73 and 2.08 % (overestimates) for the Cravero and Ruggieri and John and Rigling equations, respectively. Fortuitously then, the John and Rigling equation with the plane strain modulus yields the most accurate values of a/W. The corollary is that use of the plane strain modulus with these alternative equations leads to lower resistance (R) curves, i.e., conservative measures of the toughness, although with a systematic error of the order of two to three %.

We have seen in **Table 4** that the effective modulus is closer to plane stress than to plane strain for $0.1 \le a/W \le 0.7$. When the plane stress modulus is used to estimate the value of u, then the Cravero and Ruggieri equation yields the most accurate values of a/W. However, even in this case there remains an underestimate of average (absolute) value equal to 2.4 %, ranging from 7.09 % (underestimate) at $a/W = 0.1$ to 0.5 % (underestimate) at $a/W = 0.7$. These are not large errors, but if considered unacceptable, then it is possible to deduce values of the effective modulus that reduce the errors even further. This can be done by calculating E_{eff} values which, when used to calculate u and substituted in the Cravero and Ruggieri equation, give accurate values of a/W (compared with the FEA results). E_{eff} is a function of u (see Eq 2), and finding values of u by trial and error that give correct values of a/W for 3D plane-sided specimens when used in the Cravero and Ruggieri equation enables calculation of corresponding values of BCE_{eff}. It is then straightforward to deduce E_{eff}/E by dividing this result by the values of $BCE_{pl\sigma}$ ($= BCE$) for this geometry from the FEA calculations. Next, α may be found by rearranging Eq 5 to give:

$$\alpha = (1/\nu^2 - 1)(E_{eff}/E - 1) \tag{8}$$

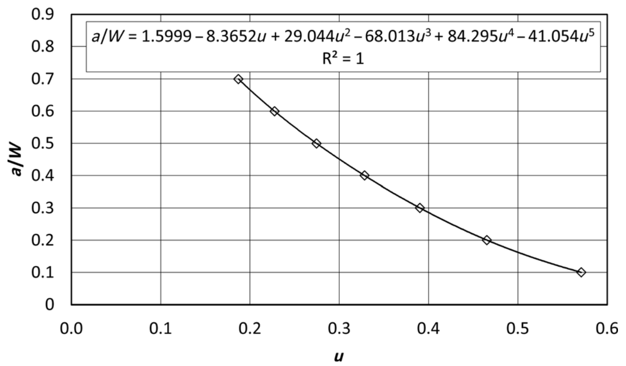
As noted above, $\alpha = 0$ and 1 for conditions of plane stress and plane strain, respectively. Next, fitting these values to a power-law equation, we find

$$\alpha = 0.2546 - 0.9873u + 3.4748u^2 \tag{9}$$

Values of E_{eff} can then be found by rearranging Eq 8 to give $E_{eff} = E(1 + (\nu^2/(1 - \nu^2)\alpha = E(1 + 0.0989\alpha)$ for $\nu = 0.3$. These values can then be used to estimate improved values of a/W from measured compliance values. In this way, it can be shown that the average error in a/W over the range $0.1 \le a/W \le 0.7$ can be reduced to 0.09 %.

Better still, for this particular clamped-specimen SE(T) test geometry ($H/W = 10$, $W/B = 1$), the FEA results for the

FIG. 1 Curve fit of Eq 10. Diamond-shaped points plot the values of u, using the definition of u (Eqs 2 and 4), at given values of a/W using the average 3D plane-sided FEA data shown in **Table 2** and the plane-stress modulus E to calculate $B_{eff}CE_{eff}$. The result of the curve-fit is the compliance equation, Eq 10, given in the figure along with the R^2 value.



compliance can be fitted directly to a compliance equation. Using the plane-stress modulus E to calculate $B_{eff}CE_{eff}$ and inserting this in the equation for u, the result of the curve-fit using the average 3D FEA data shown in **Table 2** is the compliance equation[10]:

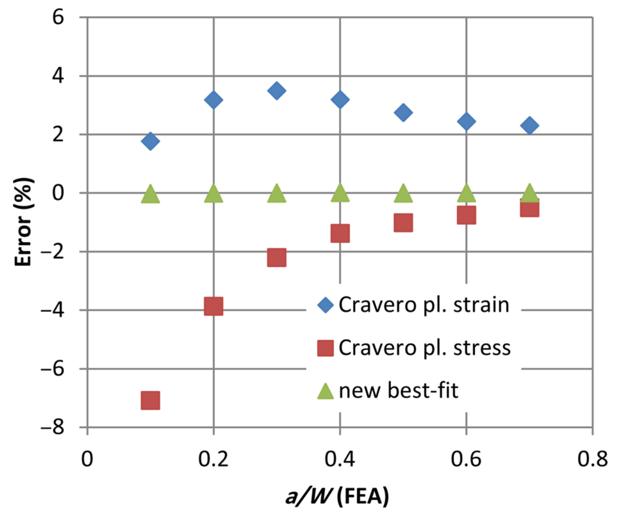
$$a/W = 1.5999 - 8.3652u + 29.044u^2 - 68.013u^3 + 84.295u^4 - 41.054u^5 \quad (10)$$

This equation yields an absolute error in a/W of 0.01 % for $0.1 \leq a/W \leq 0.7$, and would seem to be the simplest way of accurately calculating a/W from CMOD compliance, implicitly taking into account the change in constraint between shallow and deep cracks. It must be emphasized, however, that this equation is valid only for the stated conditions and should not be used for other geometries. The curve fit, with Eq 10 and R^2, is shown in **Fig. 1**.

The effect of using this "best-fit" equation can be illustrated by comparing the values of a/W calculated from Eq 1 with values calculated from the Cravero and Ruggieri equation (Eq 6), with the latter derived from the plane strain or plane stress limits of the variable u (Eq 2 and Eq 4). The errors are shown in **Fig. 2**. It is evident that the Cravero and Ruggieri equation brackets the correct a/W values when the plane strain and plane stress moduli are used, and that Eq 10 works well (as it should, since it was obtained by fitting the FEA data). The errors inherent in the plane-stress or plane-strain approximations can range between ≈−7 % and ≈+4 %, respectively, over the range of values in **Fig. 2**, but are of the order of a few % (≈±4 %) in the region of most practical interest ($0.2 \leq a/W \leq 0.6$).

[10]Note that the choice of modulus to use in calculating $B_{eff}CE_{eff}$ from the FEA data, either the plane-stress or plane-strain modulus, is arbitrary. Either will generate accurate values of a/W, although obviously requiring different coefficients in Eq 10. The plane-stress modulus is recommended because it does not involve Poisson's ratio and because it is the effective modulus used in ASTM E1820-15a [9]. Note also that the effect of side grooves is accounted for in Eq 3. In practice, the measured compliance for side-grooved specimens is adjusted to the plane-sided value using Eq 3 and then used to calculate u, and finally to calculate a/W using Eq 10.

FIG. 2 Error (% difference) between value of a/W calculated from alternative compliance equations and from direct FEA as a function of a/W. Alternative compliance equations use: Eq 10 with the plane-stress modulus, E, (triangular points); and the Cravero and Ruggieri Eq 6 in the limit of plane strain (diamond-shaped points) or plane stress (square-shaped points), i.e., with the variable u (Eqs 2 and 4) calculated using $E_{eff} = E/(1-\nu^2)$ or $E_{eff} = E$, respectively).



Discussion

This note is intended to contribute to the selection of equations to be used for calculation of crack size in a proposed toughness test standard using clamped SE(T) specimens with $H/W = 10$ and $W/B = 1$, with side grooves of total depth 10 % (5 % each side). The normal range of a/W for practical purposes will be $0.2 \leq a/W \leq 0.6$, and to ensure that this is covered adequately, the present paper has focused on the range $0.1 \leq a/W \leq 0.7$. It has been shown that over this range, the constraint condition for the CMOD compliance C lies between plane strain for shallow cracks and plane stress for deep cracks, being slightly closer to plane stress over the target range of a/W ($0.1 \leq a/W \leq 0.7$). In the literature, compliance equations are derived from FEA results, normally in the limit of plane strain, that are subsequently fitted with a power-law equation of the variable $u = 1/(1+\sqrt{(B_{eff}CE_{eff})})$. For the plane strain condition, $E_{eff} = E_{pl\varepsilon} = E/(1-\nu^2)$, and for the plane stress condition, $E_{eff} = E_{pl\sigma} = E$; a/W is the same function of u in either case provided the relevant value of E_{eff} is used in u. B_{eff} is the "effective thickness," $B_{eff} = B-(B-B_N)^2/B$ (see Eq 3), where B is the specimen thickness and B_N is the net thickness, i.e., the specimen thickness minus the total side-groove depth.

The results in this paper draw on independent FEA calculations from three laboratories. It has been shown in these calculations that the use of the effective thickness (Eq 3) for side-grooved specimens provides an accurate correction for the compliance[11], and consequently attention has been focused on plane-sided specimens (2D and 3D) to deduce the best-fitting equations. Evidently from the paragraph above, the experimental compliance will lie between plane stress and plane strain, so that to deduce a/W from the experimental compliance, a value of E_{eff} between $E_{pl\varepsilon}$ and $E_{pl\sigma}$ should be used to give accurate results. For practical purposes, it has been recommended by several authors that the plane stress modulus be used. For comparison, in the current edition of ASTM E1820 [9] the plane-stress modulus $E_{pl\sigma} = E$ is used in calculating u for SE(B) and C(T) specimens. The plane-strain value $E_{pl\varepsilon}$ was used in previous editions, presumably because the crack tip is in primarily plane strain constraint. The predominance of plane stress in the ASTM test is reinforced by the fact that for the geometry of specimens in ASTM E1820, $W/B = 2$, i.e. the specimens are thinner than for $W/B = 1$ (the geometry of interest in this work) and hence closer to plane stress than the specimens studied here. Clearly there is some ambiguity in the choice in that the constraint varies depending on the specimen geometry, and hence there is uncertainly in choosing the best value of E_{eff} to use in calculating u and a/W. The present paper has been prepared to illuminate the situation for $B \times B$ SE(T) specimens with and without side grooves.

Notably, the effective constraint is nearly plane strain for shallow cracks in $B \times B$ SE(T) specimens. This is somewhat surprising, because the bulk of the specimen is in plane stress for this condition. A similar phenomenon has been noted for SE(B) specimens [11] and explained as a reflection of the fact that for a shallow crack, the CMOD reflects the constraint condition at the crack tip, and this condition is close to plane strain.

Comparison With Experiment

There have been no experiments reported to investigate specifically the relation between measured and predicted crack size for SE(T) specimens. However, a round robin coordinated by CANMET has been carried out [14]. These results have been revisited in light of the current work. Unfortunately, the comparison proved inconclusive. The round robin used the compliance function of Shen and Tyson [4], which is practically identical to that of Cravero and Ruggieri [7], with the plane stress modulus E used as E_{eff}. In the experiments, the final measured crack size (a/W) varied between 0.44 and 0.62. Comparing calculated with measured crack size, on average the calculated crack size overestimated the measured size by about 1 %, with a standard deviation of 4.4 %. Application of the recommended compliance function (Eq 10) would have increased the overestimation still further by about 1 %, clearly an "improvement" in the wrong direction. However, the scatter is such that this result cannot be considered conclusive. It is further complicated by the fact that the experimental results required corrections for rotation and for side-groove depth, both of which introduce small uncertainties, and that crack fronts in the tests invariably showed some degree of curvature. It would have been preferable to use the initial crack size in the comparison because this would eliminate the need for a rotation correction. However, this was also not conclusive because of experimental difficulties experienced by most labs participating in the round robin in accurately measuring the initial compliance. It was found that the compliance data frequently corresponded to an apparent negative crack growth, which is obviously nonphysical. The solution was to calculate an improved estimate of the initial crack size a_{0q} following the procedure described in ASTM E1820 [9]. Values of initial crack size ranged between 0.33 and 0.46. Comparing calculated with measured crack size, the average result was that the calculated initial crack size overestimated the measured size by about 3.4 % with a standard deviation of 4.7 %, again leading to an inconclusive result.

[11] The variation of the compliance with side-groove depth is well described by B_{eff}, i.e., the compliance for plane-sided specimens is B_{eff}/B times the compliance of a side-grooved specimen.

Conclusions

1. For calculation of a/W using SE(T) specimens of square cross section ($W/B = 1$), using the plane stress modulus E in the compliance equation proposed by Cravero and Ruggieri [7] yields values (underestimates) within 7 % of the correct value over the range $0.1 \le a/W \le 0.7$ with an average error of 2.4 %. This leads to slightly non-conservative R curves in comparison with the curves that would result from use of the plane strain modulus.
2. The use of the effective thickness $B_{eff} = B - (B - B_N)^2/B$ yields values of $B_{eff}C$ that are constant for a given a/W within an average error of 0.70 % over the range $0.1 \le a/W \le 0.7$.
3. The effective modulus varies with a/W, being near plane strain for shallow cracks and near plane stress for deep cracks. Defining a parameter α (Eq 5) such that $\alpha = 0$ and 1 for conditions of plane stress and plane strain, respectively, it is found that α decreases from 0.84 at $a/W = 0.1$ to 0.29 at $a/W = 0.7$, the average over this interval being 0.47. That is, over the range of interest, the effective modulus is slightly closer to plane stress than plane strain.
4. Values of a/W calculated from the equation of Cravero and Ruggieri can be further improved by using the effective modulus $E_{eff}/E = 1 + 0.0989\alpha$ (for $\nu = 0.3$) with α given by Eq 9 and $u = 1/(1 + \sqrt{(B_{eff}CE_{pl\sigma})})$. This reduces

the absolute error over the range $0.1 \le a/W \le 0.7$ to 0.09 %.

5. By directly fitting the FEA results for 3D plane-sided SE(T) specimens with $H/W = 10$ and $W/B = 1$, a curve fit of a/W to the parameter u yields the equation for a/W given by Eq 10. This equation, valid over the range $0.1 \le a/W \le 0.7$, provides values with an average error of only 0.01 % and is the simplest way to deal with the variation in constraint between shallow and deep cracks. It must be emphasized that the modulus to be used in u is the plane stress modulus $E_{pl\sigma} = E$, and that this curve fit applies strictly only to the stated geometry, i.e., $H/W = 10$, $W/B = 1$, and $0.1 \le a/W \le 0.7$.

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